KMEP Presents

Mathemagics

Classroom Arithmetricks

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Acknowledgements

The document *Classroom Puzzles* was compile, adapted, and developed by the Kivalliq Math Education Panel Kivalliq School Operations acknowledges and thanks the following teachers for their time and commitment to this project.

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The math tricks found in this document were adapted from the following sources:

- Blum, Raymond (2000). Math Tricks Puzzles and Games. Scholastic Inc. Toronto.
- http://www.angelfire.com/me/marmalade/mathtips.html
- http://www.pen.k12.va.us/Div/Winchester/jhhs/math/puzzles/mtricks.html

Preface

Mathemagics and *Arithemtricks* are number and card tricks that capture the imagination of math students. Most of the activities have a mathematical explanation that should satisfy you, the teacher. However, even the activities that don't come with an explanation are valuable for the excitement and mystery that they generate in your class. *Mathemagics* can become special activitiy that you perform for your class throughout the year.

Use *mathemagic* to engage your students and encourage them to come up with reasons why the *arthimetricks* work or support them to develop their own variations of the tricks.

An electronic version of this document can be found in the First Class System:

Teacher Conferences→Kivalliq Math Forum→Math Month

Jim Kreuger
Baker Lake
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Counter Con

When your back is turned, a student removes some counters from a pile. After performing some mathemagic, you are able to determine the number of counters that are hidden in his or her hand!

Materials:
- 20 counters
- A small bowl

Presentation:
1. Place a bowl of 20 counters on the table, and then turn your back. Ask your student to remove any number of counters from 1 to 9 and put them in his or her pocket.
2. Tell him or her to count the number of counters that remain in the bowl.
   For example:
   He or she removes 7 counters, therefore 20 – 7 = 13 remain
3. Ask her to find the sum of the digits of that number.
4. Tell her to remove that number of counters from the bowl and put those in his or her pocket too.
   For example:
   13 remain → 1 + 3 = 4; four more counters are added to his or her pocket.
5. Ask him or her to remove any number of counters from the bowl and hide them in his or her hand.
6. When you turn around, take one of the counters out of the bowl. Hold it to your forehead and pretend to be in deep thought for a few seconds. Then reveal the number of counters that your student is hiding in her hand!

How to Do It:
When you turn around, secretly count the number of counters that remain in the bowl. Just subtract that number from 9. That difference is the number of counters that she is hiding in her hand!

9 - 3 counters in the bowl = 6 counters in her hand

An Exception:
If your student hides 9 counters in her hand and there are no counters left in the bowl, hold the bowl to your forehead.

The Secret
This trick uses a mathematical procedure called **<u>casting out nines</u>**. The first subtraction results in a number between 10 and 20. Any number between 10 and 20 minus the sum of its digits always equals 9.

11	11 - (1 + 1) = 9
12	12 - (1 + 2) = 9
13	13 - (1 + 3) = 9
14	14 - (1 + 4) = 9
15	15 - (1 + 5) = 9
16	16 - (1 + 6) = 9
17	17 - (1 + 7) = 9
18	18 - (1 + 8) = 9
19	19 - (1 + 9) = 9

Table 1: Casting out 9s

A Variation:
You can also tell how many counters are in his or her pocket. It will always be 11!

Pencil Memory

Your student randomly chooses any 3-digit number, and then performs some mathemagical calculations with a pencil. When the pencil is sharpened and the shavings rubbed on your arm, the final total mysteriously appears as if the pencil remembered your number!

Materials:

- Pencil
- Paper
- Pencil Sharpener

Preparation:

Using liquid soap, write the number 1089 on the inside of your lower arm. Let your arm dry so that it is invisible to the eye.

Presentation:

1. Ask a student to write any 3-digit number on a piece of paper without letting you see it. The first digit and the last digit must differ by at least two.
2. Ask him or her to reverse the 3 digits and write this new number below the first number. Now ask him or her to subtract the smaller of the two numbers from the larger of the two and record this difference on the paper and circle it seven times with the pencil.
3. Ask him or her to reverse the digits of the circled number and record this new number on the paper. Add this new number to the circled number, record the sum on the paper and circle it seven times with the pencil.
4. Instruct the student turn their paper over and quickly sharpen their pencil by turning the handle of the sharpener seven times.
5. Tell your student that their pencil will remember the math that it has performed and that the pencil shavings now have the answer to his or her calculations. Take some shavings in your hand and rub them on your arm that has the saop writing. Like magic, the number 1089 mysteriously appears!

The Secret:

It does not matter which 3-digit number your student begins with. If the math is done correctly, the final total will always be 1089!

For Example
652 → 256
652 – 256 = 396
396 → 693
396 + 693 = 1089

Mental Mathemagics

Initially these *arithmetrics* may be used to amaze your class, but they should also be taught as mental math strategies to assist students in problem solving.

Mental Multiplication I (The Rule of 10)

You and your class should already know the Rule of 10, but for the sake of completeness, It is include here. Simply put, the Rule of 10 states that to multiply by 10, just add a 0 behind the number.

For example:
6 x 10 = 160   16 → 160

Mental Multiplication II (The Rule of 11)

Multiplying any two-digit number by 11 is easy to do in your head, if you know this *arithmetric*.

For Example:

1. Take 11 x 36.
2. We are only concerned with the second number. (36)
3. Separate the two digits in you mind and focus on the space between them. (3___6)
4. Add the two digits together and put the resulting sum in the space between the two digits. (3 + 6 = 9 → 3 _9_ 6)
5. That's it! 11 x 36 = 396

Variation:
If the sum of the two digits is greater than 9, additional steps are required.

For Example:

1. Take 11 x 76 (7____6).
2. Add the two digits together and place the ones digit from the sum into the space between the two digits. (7 + 6 = 13 → 7 _3_ 6).
3. Now take the tens digit from the sum and carry it over to the hundreds place. (736 → 836).
4. That's it! 11 x 76 = 836

Mastery of the Rule of 11 takes a little time, so practice it on paper first!

Mental Multiplication III (11 – 19)

If you can multiply and know your facts up to 10 x 10, you will some be able to amaze your students with your mental *mathemagics*. With this *arithmetric* and a little practice, you will be able to multiply any two numbers from 11 to 19 in your head quickly, without the use of a calculator**.**

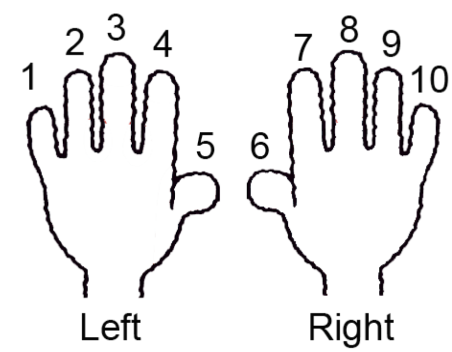
For Example:

1. Take 17 x 14.
2. In your mind, always place the larger number on top.
3. Mentally note the top number and the one's digit of the bottom (17 and the 4 from the 14). These are the only numbers that you will need!
4. First add together the top number and the one's digit of the bottom (17 + 4 = 21)
5. Now multiply this sum by 10--add a zero behind it. (21→ 210)
6. Multiply the one's digits by each other, ie the 4 and the 7. (4 x 7 = 28)
7. Finally, add the number from step 5 to the number from step 6 to get your answer. (210 + 28 = 238!)

17 x 14 = 238

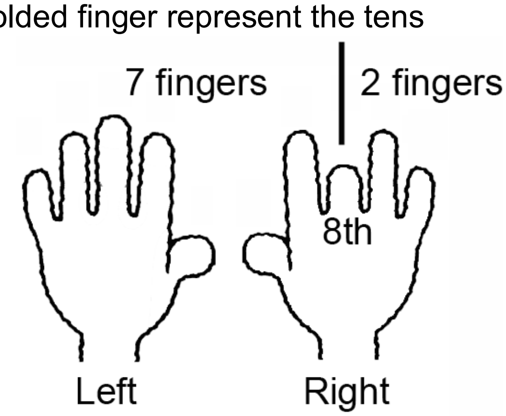
That is all there is to it. Now practice it on paper and in about five minutes you will be ready to amaze your class!

Mental Multiplication IV (The Rule of 9)



This trick is more physical than mental and involves using your fingers on both hands. To multiply any single digit by 9, try this simple *arithmetrick*:

1. Spread your two hands out and place them on a desk or table in front of you. Starting on the left, each finger represents a number from 1 to 10.
2. To multiply 9 by a number, fold down the finger that represents the number you are multiplying. To multiply by 2, it would be the 2nd finger to multiply by 4 it would be the 4th finger and so on.
3. Once the finger is folded, your answer is determined by the position of the remaining fingers. The fingers to the left of the folded finger represent the tens digits and the fingers to the right of the folded finger represent the ones digits.



For Example:
9 x 8 → bend your 8th finger and count the fingers to the left and right of the bent finger. You have seven fingers to the left and 2 fingers to the right, therefore your answer is 72!

This *arithmetrick* works for anything up to 9x10!

Mental Multiplication V (The Rule of 5)

This simple *arithmetrick* replaces multiplying a number by 5 with dividing a number by 2 and then multiplying by 10 (add a zero or move the decimal place). With practice, this can be done quickly in your head.

For Example:
5 x 244 → (244 ÷ 2) x 10 → 122 x 10 → 1220

Number Arithmetricks

These *arithmetrics* may be used to stump and amaze your class, but, with patience and persistence, they should be able to come to understand them.

Phone Number Redial

Your students begin with their phone numbers, and then perform some mathemagical calculations on a calculator. When they are done, they are right back where they began.

Materials:

- Calculator

Presentation:

1. Key in the first three digits of your phone number (<u>not the area code</u>)
2. Multiply by 80
3. Add 1
4. Multiply by 250
5. Add the last 4 digits of your phone number
6. Add the last 4 digits of your phone number again.
7. Subtract 250
8. Divide number by 2

Do you recognize the answer?

Some Sum

Which sum is greater?

987654321		123456789	
87654321		12345678	
7654321	or	1234567	
654321		123456	
54321		12345	Believe it or not, the sums are identical.
4321		1234	Both add up to 1,083,676,269! Have your
321		123	students verify the answer for themselves.
21		12	Ask them to suggest reasons why the
+ 1		1 +	sums are the same.

The Number 3 Trick

Perform this *mathemagic* and always end up with the number 3. Knowledge of this *arithmetrick* can be used often by a creative math teacher.

1. Take a number.
2. Double it.
3. Add 9.
4. Subtract 3.
5. Divide by 2.
6. Subtract your original number.
7. Your answer should be 3.

The Secret

Here is the algebra to explain it:

Take a number:	Let x = the number
Double it:	2x
Add 9:	2x + 9
Subtract 3:	2x + 6
Divide by 2:	x + 3
Subtract your original number:	3

That's why your answer is 3.

The Number 4 Trick

Perform this *mathemagic* and always end up with the number 4. This too *arithmetrick* can be put to good use by a creative math teacher.

1. Take a number.
2. Double it.
3. Add 12.
4. Subtract 4.
5. Divide by 2.
6. Subtract your original number.
7. Your answer should be 4.

The Secret

Here is the algebra to explain it:

Take a number:	Let x = the number
Double it:	2x
Add 12:	2x + 12
Subtract 4:	2x + 8
Divide by 2:	x + 4
Subtract your original number:	4

That's why your answer is 4.

The Number 5 Trick

This arithmetrick is especially good for grade 3.

1. Pick a secret number
2. Double it.
3. Multiply by 5.
4. Ask for the total.
5. Whatever the total is, knock off the last digit and you will have the secret number.

The Secret

Here is the algebra to explain it:

Take a number:	Let x = the number
Double it:	2x
Multiply by 5:	10x

10 multiplied by any number just adds a zero to the ones place.

The Amazing Number 37

The number 37 is an amazing number for the following reasons:

37 x 3 =	111
37 x 6 =	222
37 x 9 =	333
37 x 12 =	444
37 x 15 =	555
37 x 18 =	666
37 x 21 =	777
37 x 24 =	888
37 x 27 =	999

The Secret

The above facts are actually the result of the first fact, namely

37 x 3 = 111

The remaining facts all derive from it.

37 x 3 = 111	
37 x 6 = 37 x 3 x 2 = 111 x 2 =	222
37 x 9 = 37 x 3 x 3 = 111 x 3 =	333
37 x 12 = 37 x 3 x 4 = 111 x 4 =	444
37 x 15 = 37 x 3 x 5 = 111 x 5 =	555
37 x 18 = 37 x 3 x 6 = 111 x 6 =	666
37 x 21 = 37 x 3 x 7 = 111 x 7 =	777
37 x 24 = 37 x 3 x 8 = 111 x 8 =	888
37 x 27 = 37 x 3 x 9 = 111 x 9 =	999

Mysterious Number

Follow the directions below to discover a mysterious number.

Materials:

- Calculator

Presentation:

Ask your students to perform the following calculations:

1. Enter 999999 into your calculator, then divide it by seven. The result will be a mysterious number!
2. Throw a die (or randomly pick a number from 1 to 6) and multiply the result by the mysterious number.
3. Arrange the digits of the product from lowest to highest from left to right to form a six-digit number.
4. What is the number?

The Secret

999999 divided by 7 yields the mystery number 142857. Multiplying this mystery number by any number from 1 to 6 will produce a number with the same six digits in possibly a different order.

Number Names

Here is an *arithmetrick* that relies on spelling and a curious property.

Presentation:

1. Think of any number from 1 through 100.
2. Write down its name.
3. Count the number of letters in its name to obtain a second number.
4. Count the number of letters in the second number to obtain a third number.
5. Continue in this way until the chain of numbers ends on a number that keeps repeating.
6. What is the number?

The Secret

Four is the only number that contains it number of letters in its name. The activity continues until you come to a number with four letters, the next number will be four and you will then be stuck with four.

Magic 8-Call

Presentation:

1. Take your phone number (treating it as a seven-digit number) and multiply it by 8.
2. Then write down the following three numbers:
   a) your phone number,
   b) 8, and
   c) the product of your phone number and 8.
3. Add up all the individual digits in those three numbers.
4. If the sum is more than one digit, take that sum and add up its digits.
5. Continue adding up digits until only one digit is left.
6. What is the digit?

The Secret

This *arithemtrick* works for any number not just seven digit phone numbers. The answer can be proved using algebra.

Let x be your phone number, therefore 8x is the product of your phone number and 8. the sum of your phone number, 8 and the product of your phone number and 8 becomes:
$x + 8 + 8x = 9x + 8$

We know that 9 times any digit give predictable results (see Finger Math), that is:
9x will produce a two-digit number with a tens digit of x-1 and a ones digit of 10 – x.

Our final sum becomes:
$(x - 1) + (10 – x) + 8 = 9 + 8 = 17 \rightarrow 1 + 7 = 8$

Time Travel

With a little theatrics, you can use arithmetricks to “prove” to your students that you can time travel.

Materials:

- Index card
- Envelope
- Pencil and paper

Preparation:

On an index card, write “There are no orange kangaroos in Denmark!” and seal it in an envelope.

Presentation:

Tell your class that you can time travel and that you have already asked them a question in the future, wrote down their answer, and traveled back in time with that answer in a sealed envelope. You now want to ask them the question and verify their answer with the one from the future. This set-up should get everyone’s attention!

Ask the class to crowd around one desk and work together to solve the following questions:

1. Think of a number from 1 to 10
2. Multiply it by 9
3. Add the digits together
4. Subtract 5 and circle your answer seven times
5. Find the letter of the alphabet which corresponds to your circled number (1 = A, 2 = B, 3 = C...)
6. Think of a country that starts with that letter and write it down.
7. Circle the last letter in the country seven times and then think of an animal that starts with this letter. Write down the name of the animal.
8. Circle the last letter in the name of the animal seven times and think of a color that starts with this letter. Write down the name of this colour.
9. Fold your paper up and let everyone touch the paper.

Now with great fanfare produce the sealed envelope. Open it in full view of your class, read it with a puzzled look and state “There are no orange kangaroos in Denmark!”

The Secret:
This *arithmetrick* is based on the probability of most common responses. The first four steps is some mathemagic that always results in the answer of 4. This means that your students will be looking for a country that begins with the letter “D”. There are only four “D” countries in the whole world (Dominican Republic, Dominica, Djibouti, and Denmark) and Denmark is the most well-known and therefore the most popular choice. Denmark ends in a “K” and most students will think of a kangaroo for their “K” animal. Kangaroo ends in the letter “O” and orange is the only colour that begins with an “O”.

This trick does not always work, as koala bear is another well-known “K” animal.

Variation:
If your students are too young to know world geography, try this one.

Ask the class to crowd around one desk and work together to solve the following questions:

Take a number: Let x = the number
Double it: 2x
Add 9: 2x + 9
Subtract 3: 2x + 6
Divide by 2: x + 3
Subtract your original number: 3
That's why your answer is 3.

1. Think of a number from 1 to 10

2. Double it.
3. Add 9.
4. Subtract 3
5. Divide by 2.
6. Subtract your original number.
7. Add 12 and circle your answer seven times.
8. Find the letter of the alphabet which corresponds to your circled number (1 = A, 2 = B, 3 = C...)
9. Think of a province in Canada that starts with that letter and write it down.
10. Circle the last letter in the province seven times and then think of a colour that begins with this letter. Write down the name of the colour.
11. Circle the last letter in the colour seven times and think of an animal that starts with this letter. Write down the name of this animal.
12. Fold your paper up and let everyone touch the paper.

Now with great fanfare produce the sealed envelope. Open it in full view of your class, read it with a puzzled look and state “There are no orange elephants in Ontario!”

Missing Cards Trick

This card trick is impressive and will keep your students guessing until they figure out what you are doing.

Materials:

- Deck of cards

Presentation:

1. Hand your student an ordinary deck of 52 playing cards. Ask him or her then to secretly remove three cards from the deck, except that no two of the three cards may add up to 10. (For example two of the cards may not be a three and a seven.)
2. Tell him or her that you will now determine the values of these three cards simply by removing cards from the deck quickly, emphasizing that you will do so without counting or memorizing any cards.
3. For purposes of this trick, we consider each card solely according to the face value, with each ace as 1 and each picture card as 10. Thus, for example, six of clubs is just 6, ten of clubs is just 10, and queen of hearts is just 10.
4. Have your student lay out the cards, in no particular order, in any convenient fashion, such as a 7x7 array. (The random order is so the removal of cards won't make it obvious what was taken.)
5. Now, remove cards in pairs that sum to 10 or 20, such as four of spades and six of diamonds, or ace of clubs and nine of hearts, or ten of hearts and queen of spades.
6. When you have done this as much as possible. you will be left with either 1 or 3 cards.

The Secret:

1. If there is one card left, with value x, then $x < 10$ and two of the picked cards have value 10 and the third has value $10 - x$.
2. If there are three cards left with values x, y, and z each under 10, then the cards chosen have values $10 - x$, $10 - y$, $10 - z$.
3. However, it is possible that one of x, y, and z has value 10, say x. In this case, the three card values are 10, $10 - y$, $10 - z$. (For instance, if there are three cards left, a ten, a two, and a five, then the three cards removed have values 10, 8, and 5.)

Nine-Card Trick

This card trick is easy to learn and is sure to amuse your students for a long time.

Materials:

• Standard deck of 52 cards

Presentation:

1. Take any nine cards from a standard 52-card deck and deal the nine cards into three piles of cards each.
2. Pick up any pile and remember its bottom card. Drop this pile on top of one of the others, and these six on top of the last pile.
3. The cards remain down throughout.
4. Hold the nine cards down and spell the value of the chosen card by dealing one card down to the table for each letter. So, if the card chosen is the Queen of Spades, spell Q U E E N and drop the remaining cards on top. Pick up the packet and spell O F. As before, drop the rest of the cards on top. Pick up the packet and spell the suit of the card you selected: S P A D E S. Drop the rest of the packet on top.
5. The position of the chosen card should now be at a random spot because card names spell out with as few as 10 letters up to as many as 15.
6. So, now spell M A G I C and turn over this last card. Believe it or not, this will always be the selected card!

The Secret:

This is a famous magic trick developed by an American magician, Jim Steinmeyer. I could not find any mathematical explanation so maybe it is mathemagical?

X-Ray Vision

You can prove to your students that you have mathemagical powers and can see through solid objects and add the bottoms of 5 dice!

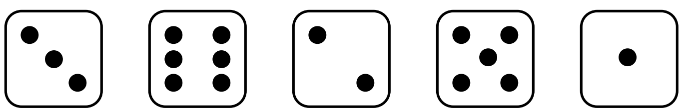
Materials:

- 5 dice

Presentation:

1. Tell your students that you are going to look through the dice and find the sum of the bottom numbers.
2. Throw 5 dice on the table.
3. Pretend that you can see through the dice all the way down to the bottom numbers. (What you are really doing is finding the sum of the top numbers.)
4. Announce the total of the 5 bottom numbers. (Just subtract the sum of the top numbers from 35.) Then carefully turn over the 5 dice and add the bottom numbers. Your students won't believe their eyes!

For Example



The sum of the top numbers is 17, therefore the sum of the bottom numbers is 35 – 17 = 18!

The Secret:
On any die, the sum of the top number and the bottom number is 7. So, if 5 dice are thrown, the total of all the top numbers and bottom numbers is:

5 x 7 = 35
Dice Total

A Variation:
Use a different number of dice and subtract the sum of the top numbers from a different number other than 35. To find that number, just multiply the number of dice by 7.

Ask your students to try to figure out how you do the trick.

Lucky Number Seven

Tell your students that you have the luckiest calculator in the world. No matter which number they enter, it is magically transformed into the lucky number 7!

Materials:

- A calculator

Presentation:

Hand your student your calculator and have him or her:

1. Enter any number that is easy to remember and is 8 digits or less.
2. Double that number.
3. Subtract 16 from that answer.
4. Multiply that result by 4.
5. Divide that total by 8.
6. Add 15 to that answer.
7. Subtract her original number from that result.

For Example
54321 x 2 = 108,642
108,642 - 16 = 108,626
108,626 x 4 = 434,504
434,504 ÷ 8 = 54,313
54,313 + 15 = 54,328
54,328 – 54321 = 7

This trick can be repeated several times with the same student. No matter which number he or she starts with, the final answer will always end up "lucky"!

The Secret:

All of the tricks in this chapter were written using a branch of mathematics called algebra. In this trick, if all of the operations are carefully performed, your friend's original number is eliminated. Adding 15 in Step 6 guarantees that the final total will always be lucky number 7.

A Variation:

Experiment by adding a different number in Step 6 and the final total will be a different number.

PAIR-A-DICE

Your student rolls 2 dice when you are not looking. After he or she performs a few calculations on a calculator, you are able to determine the two numbers rolled on the dice!

Materials:

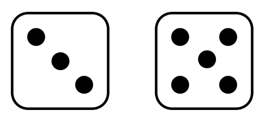
- 2 dice
- A calculator
- Paper and pencil

Presentation:
When your back is turned, have a student:

1. Roll two dice.
2. Multiply the top number on the first die by 5, using a calculator or paper and pencil.
3. Add 12 to that answer.
4. Double that total.
5. Add that result to the top number on the second die
6. Add 15 to that answer.

Finally, ask your student to hand you the calculator with his or her final total still in the display. Just subtract 39 from this total the difference will be a two digit number, with the tens digit representing the first die and the ones digit representing the second die.

For Example:
3 x 5 = 15
15 + 12 = 27
27 x 2 = 54
54 + 5 =59
59 + 15 = 74



74 – 39 = 35 → The first die was 3 and the second die was 5.

The Secret:
Multiplying by 5 and then doubling is just like multiplying by 10. This puts the number on the first die in the tens place. Adding the number on the second die puts that number in the ones place. Every other operation is mathematical hocus-pocus and adds an extra 39 to the total. Subtracting this 39 reveals the two top numbers on the dice.

Once your students figure out this arithmetrick, have them invent their own!